

Axion-dilaton Gauged S -duality and its Symmetry Breaking

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The equation of the axion-dilaton gauge potential is derived for gauged S -duality on general type IIB string vacua. Based on this formulation of S -duality, we show how the vacua spontaneously break gauged S -duality, leading to a vanishingly small cosmological constant.

0. In this article we derive the equation for the gauge potential of gauged S -duality[1] on moduli curves in the scalar part of type IIB string universal hypermultiplet. When we consider the space of coupling constant as the Poincaré upper half plane \mathfrak{h} , the moduli curve for gauged S -duality after gauge fixing, is a torus $\Sigma = \Gamma \backslash \mathfrak{h}$ where $\Gamma = SL(2, Z)$.

The two dimensional gauge potential \mathcal{A}_i , $i = 1, 2$ on moduli curve Σ with Ramond-Ramond (R-R) fluxes has the following characteristics.

1. It has three variables: the coupling constant g and vector R-R and Neveu-Schwarz–Neveu-Schwarz (NS-NS) fluxes on Σ . The fluxes f_1 and f_2 are the integrations of the Poincaré length 1-form f along two independent geodesics C_1 and C_2 on \mathfrak{h} . These geodesics are the fundamental classes on Σ and are two circles connecting fixed points of two elements, the S -duality element γ and the identity 1, of Γ .
2. The energy conservation law for gauge modes \mathcal{A}_i is written in terms of the total static mass of an $SL(2, Z)$ multi-plet for gauge potential \mathcal{A}_i . The square of the mass is an eigenvalue of the Laplacian Δ on Σ and the gauge potentials \mathcal{A}_i for $i = 1, 2$ are eigenstates.

The Γ -gauge symmetry on the moduli curve Σ is closely related to the geometry of Σ . Indeed the fundamental group of the moduli curve Σ is determined by the gauge group Γ and we can induce a metric on it.

1. In the following, we derive the equation of the dilaton part of the Γ -gauge potential $\mathcal{A}_\varphi = \mathcal{A}_1$ on the moduli curve Σ . For the operator acting on this, we need to sum the mass operators for the dilaton φ , the axion χ , the R-R flux f_1 and the NS-NS flux f_2 because they constitute a single S -duality multi-plet.

First, we start from the Laplace equation for a massless dilaton φ induced by the gauge potential \mathcal{A}_φ on the moduli curve Σ . The dilaton vacuum expectation value $\langle \varphi \rangle$ is associated with coupling constant g by $\sqrt{-1}e^{-\langle \varphi \rangle} = g$ when we ignore the R-R field of the D-instanton, i.e., axion. The Laplace equation on Σ for the variable g

$$\Delta \mathcal{A}_\varphi(g) = 0 \quad (1)$$

is the equation for \mathcal{A}_φ when the flux variables f_1 and f_2 are fixed.

The square root of the Laplacian Δ for the dilaton part of the potential is $\partial/\partial g$ and is the total mass operator for the gauge mode \mathcal{A}_φ .

For general type IIB string vacua, we will also need to consider the mass contributions from the other components of the S -duality multiplet. So we need to introduce the R-R (or the S -dual NS-NS) flux variables f_1 and f_2 in addition to the coupling constant g in the equation for \mathcal{A}_φ . We assume that the variables f_1 and f_2 appear in the Fourier modes of gauge potential as

$$\mathcal{A}_\varphi \sim \mathcal{A}_\varphi^0 \int \frac{dQ_1 dQ_2}{2\pi} A(Q_1, Q_2) \times \exp \sqrt{-1} \left(\sum_{i=1}^2 f_i (\text{flux}_i \text{ on } \Sigma) \times Q_i (\text{mass}_i) \right) \quad (2)$$

where \mathcal{A}_φ^0 does not depend on f_1 or f_2 .

We interpret the masses Q_i as R-R and NS-NS charges of axion and dilaton in an $SL(2, Z)$ multi-plet. By assuming a gauge potential of this form, we induced the S -duality multiplet in the R-R flux part by the action of $SL(2, Z)$ on the inner product $\vec{f} \cdot \vec{Q}$.

For the operator that acts on \mathcal{A}_φ , we consider

$$\sum_{\gamma_1, \gamma_2} \gamma^{-1} \frac{\partial}{\partial f_\gamma} \gamma \quad (3)$$

where γ_1 and γ_2 are the S -duality and identity elements of the discrete S -duality gauge group Γ . The action of γ_1 is defined as follows. In the $SL(2, Z)$ multiplet we need simultaneously to transform the integral domain geodesics $C_i \in \pi_1(\Sigma, p)$ with $p \in \Sigma$ by $\gamma_1 :^t (C_1, C_2) \rightarrow^t (C_2, -C_1)$ when we change the coupling constant g to $-1/g$. As a result of this definition, γ_1 and γ_2 select R-R and NS-NS fluxes for axion and dilaton in the gauge mode \mathcal{A}_φ .

Finally, on the moduli curve Σ , we consider the mass of an axion as the soliton mass of the non-perturbative vacuum. Its mass depends on the coupling constant such that, for an axion, strong coupling gives a light mass and weak coupling gives a heavy mass. We need to add to the mass term of the $SL(2, Z)$ multi-plet the mass of an axion given by

$$-\frac{\sqrt{-1}}{g}. \quad (4)$$

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As the result, the equation (the definition) of the gauge potential $\mathcal{A}_\varphi(g, f_1, f_2)$ is extended from Eq. 1 to the following energy conservation law for the gauge mode

$$\left(\frac{\partial}{\partial g} - \sum_{i=1}^2 \sigma_{\gamma_i} \left(\frac{\partial}{\partial f_i} \right) + \frac{\sqrt{-1}}{g} \right) \mathcal{A}_\varphi(g, f_1, f_2) = 0. \quad (5)$$

Where we put $\sigma_\gamma(x) = \gamma^{-1}x\gamma$ and the actions γ_1 and γ_2 to gauge potential are defined by

$$\begin{aligned} \gamma_1 \mathcal{A}_\varphi(g, f_1, f_2) &= \mathcal{A}_\varphi(-g^{-1}, f_2, -f_1), \\ \gamma_2 \mathcal{A}_\varphi(g, f_1, f_2) &= \mathcal{A}_\varphi(g, f_1, f_2). \end{aligned} \quad (6)$$

We set the initial condition of Eq. 5 to ensure the consistency with Eq. 1

$$\mathcal{A}_\varphi(g, 0, 0) = \mathcal{A}_\varphi^{\text{Eq. 1}}(g) \quad (7)$$

where $\mathcal{A}_\varphi^{\text{Eq. 1}}(g)$ is a solution of the Laplace equation (Eq. 1). Due to the existence of γ_1 , Eq. 5 is a non-linear equation for \mathcal{A}_φ .

2. Now, based on Refs. 2 and 3, we discuss the relation between the problem of the vanishingly small cosmological constant and the above formulation of gauged S -duality. In particular we show how the type IIB string vacua break Γ -gauge symmetry spontaneously.

It is clear what kind of object we should introduce in order to spontaneously break the Γ -symmetry to the symmetry of a Fuchsian subgroup of degree one Γ_h . The subscript h labels the genus of the Riemann surface $\Sigma_h = \Gamma_h \backslash \mathfrak{h}$. The Γ -symmetry is gravitational on the Poincaré metric of \mathfrak{h} . So this object is the so-called gravitational Higgs variable on \mathfrak{h} . In our case, the Higgs variables as R-R and NS-NS fluxes f_1 and f_2 are introduced by including a cosmological constant Λ_{ptl} that is a function of Higgs variables f_1 and f_2 by analogy with Higgs potential in the action.

When the vacuum has a source term for the Lorentz metric, $\Lambda \eta_{\mu\nu}$, in the equation of motion for type IIB supergravity, the on-shell gauge group Γ is broken.[2] The Goldstone mode $\mathcal{A}_\varphi^{G(\gamma)}$ of this broken symmetry, corresponding to an element γ of the coset Γ/Γ_h , parametrizes

the space of broken symmetries. The gauge mode $\mathcal{A}_\varphi(f_1, f_2)$ that moves by the action of Γ_h around the classical field $\mathcal{A}_{\varphi,0}$ generates a non-zero, positive contribution to the cosmological constant Λ_{ptl} , which is the value of Higgs potential at the dominant variables. And two flux variables f_1 and f_2 are the lengths of two geodesics on \mathfrak{h} as fundamental classes on Σ_h . Λ_{ptl} can be considered to be the almost vanishing vacuum energy as we can see by the following argument.

The massive mode, arising from the spontaneous breakdown of the gauge group Γ to Γ_h , gives a positive contribution to the cosmological constant Λ^+

$$\Lambda^+ = \Lambda_{ptl}(f_1, f_2) + \Lambda_{mass}, \quad (8)$$

where the contribution from the gauge mode is an eigenvalue of the Laplacian Δ_{Γ_h} on Σ_h , namely the square of the mass of the gauge mode. That is

$$\Lambda_{mass} = \text{Spec} \Delta_{\Gamma_h}. \quad (9)$$

The first term in Eq. 8 (Λ_{ptl}) survives under the covariance of the remaining Γ_h -gauge symmetry.

The second term in Eq. 8 (Λ_{mass}) should cancel with the negative vacuum energy of the type IIB supergravity vacuum (for example, a de Sitter vacuum or even a broken supersymmetry vacuum) since the Goldstone mode \mathcal{A}_φ^G recovers the broken S -duality.

The scheme presented here clarifies the idea that S -duality as exact Γ -(gauge) symmetry in type IIB string vacua can be used to resolve the problem of the vanishingly small cosmological constant, as first proposed by Kar, Maharana and Singh.[3] In this interpretation, we can also see that vacua with negative vacuum energies in type IIB supergravity are in a Higgs phase of gauged S -duality.

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